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Quantum annealing-driven branch and bound for the single machine total weighted number of tardy jobs scheduling problem

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ABSTRACT

In the paper we present a new approach to solving *NP*-hard problems of discrete optimization adapted to the architecture of quantum processors (QPU, Quantum Processor Unit) implementing hardware quantum annealing. This approach is based on the use of the quantum annealing metaheuristic in the exact branch and bound algorithm to compute the lower and upper bounds of the objective function. To determine the lower bound, a new method of defining the Lagrange function for the dual problem (the generalized discrete knapsack problem) was used, the value of which is calculated on the QPU of a quantum machine. In turn, to determine the upper bound, we formulate an appropriate task in the form of binary quadratic programming with constraints.

Despite the fact that the results generated by the quantum machine are probabilistic, the hybrid method of algorithm construction proposed in the paper, using alternately a CPU and QPU, guarantees the optimal solution. As a case study we consider the *NP*-hard single machine scheduling problem with minimizing the weighted number of tardy jobs. The performed computational experiments showed that optimal solutions were already obtained in the root of the solution tree, and the values of the lower and upper bounds differ by only a few percent.

1. Introduction

The concept of quantum computing originates from the 1980s. Currently, quantum computers are available that represent one of two approaches to quantum computing. The first, offered by companies such as Google, Honeywell, IBM and Intel, are quantum computers with quantum gate models (e.g., Hadamard and Toffoli). Unlike many classical logic gates, quantum logic gates are reversible. Programming in this model of quantum computing is still a great challenge due to the small scale of solvable problems and the lack of a high-level approach, adequate to high-level languages in the programming of classic silicon computers. The second approach, called *quantum annealing*, by using effects known as quantum fluctuations and quantum tunneling, determines the best possible solution to the optimization problem, see e.g., (Bozejko et al. [1]). D-Wave Systems provides quantum computers, proposing an approach to computation limited only to the use of quantum annealing; however, it is perfectly suited to the needs of the discipline of operations research. In this case, instead of expressing the algorithm solving a given problem in the form of quantum gates, the user presents it as a binary quadratic programming problem. However, Aharonov et al. [2] show that quantum annealing (as an adiabatic quantum computation) is equivalent to the standard quantum-gate model of a quantum computer.

There are several attempts to apply the methodology of quantum computations to operational research problems. Regarding branch and bound quantum modifications, Montanaro [3] describes a quantum algorithm that can accelerate classical branch-and-bound algorithms near-quadratically in a general setting. The author considers a spin model addressed using branch-and-bound, which is the Sherrington–Kirkpatrick model, which is the family of classical Hamiltonians $H(x) = \sum_{1\le i<j\le n} a_{ij}x_ix_j$ where $x \in \{\pm 1\}$ and a_{ij} are distributed according to the normal distribution. Finding the lowest-energy state for such a Hamiltonian can be achieved in time $O(2^{0.5n}poly(n))$ using Grover's algorithm, which is less efficient than the approach proposed in [3] with the time $O(2^{0.226n})$.

The paper of Markevich and Trushechkin [4] shows the theoretical branch and bound for a quantum-gates based quantum computer (without experiments).

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